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Q.1 Define subgroups of a Group ~~and give an example:~~

Soln: Subgroups of a group:→ Let $(G, *)$ be a group and H is any subset of G such that $H \neq \phi$. Then by properties of a group

$\forall\ a, b \in H \Rightarrow a, b \in G$, for $H \subset G$

$\Rightarrow a * b \in G \Rightarrow a * b \in H$ or $a * b \notin H$.

If $a * b \in H$, then we say that H is stable for the composition in G and the composition in G has induced a composition in H. Now, there are two possibilities.

(I) H is itself a group with respect to the operation $*$.

II. H is not a group with respect to $*$.

Q.2. Define subgroups and give an example.

Soln:→ Subgroups:→ A non-empty subset H of a group G is called a subgroup of G if H itself a group with respect to the operation defined in G. For example.

$[\{1, -1\}, *]$ is a subgroup of $[\{1, -1, i, -i\}, *]$

and $(Z, +)$ is a subgroup of $(Q, +)$

Also, $(Q, +)$ is a subgroup of $(R, +)$.

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Q.3. Let H be a non-empty subset of a Group G. Then H is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G.

Proof: There are two conditions arise in this question

I Necessary conditions:→ Let us first suppose H is a subgroup of G and $a, b \in H$. Since H is a group, each element of H must have its inverse in H. Thus, if $b \in H \Rightarrow b^{-1} \in H$ and then by closure property $ab^{-1} \in H$. This proves the necessary condition.

II Sufficient Condition:→ Conversely, let H is a subset of G for which $a, b \in H \Rightarrow ab^{-1} \in H$. We have to prove that H is a subgroup of G, we must verify that H is closed, the identity element $e \in H$, every element of H has an inverse in H and the associative law holds for elements of H.

Let $b = a$, then we see that $a \in H \Rightarrow aa^{-1} \in H \Rightarrow e \in H$.
$\Rightarrow$ identity element of G also belongs to H.

Now, for the element e and b of H, we have $ea^{-1} \in H$ and so $b^{-1} \in H$, since b is an arbitrary element of H, we see that for any $b \in H$, $b^{-1} \in H$

Now, $a, b \in H \Rightarrow ab^{-1} \in H$

$\Rightarrow a(b^{-1})^{-1} \in H$ [By hypothesis]

$\Rightarrow ab \in H$

$\Rightarrow$ H is closed.

Finally, since the associative law does hold for H, it also holds for H which is a subset of G.

$\Rightarrow$ $(H, *)$ is a subgroup.